



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**NATIONAL  
SENIOR CERTIFICATE/  
NASIONALE  
SENIOR SERTIFIKAAT**

**GRADE/GRAAD 12**

**MATHEMATICS P2/WISKUNDE V2**

**NOVEMBER 2015**

**MEMORANDUM**

**MARKS: 150**  
**PUNTE: 150**

**This memorandum consists of 27 pages./  
Hierdie memorandum bestaan uit 27 bladsye.**

**NOTE:**

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.
- Penalty of only 1 mark for incorrect rounding throughout the paper (Q1.2.1)

**LET WEL:**

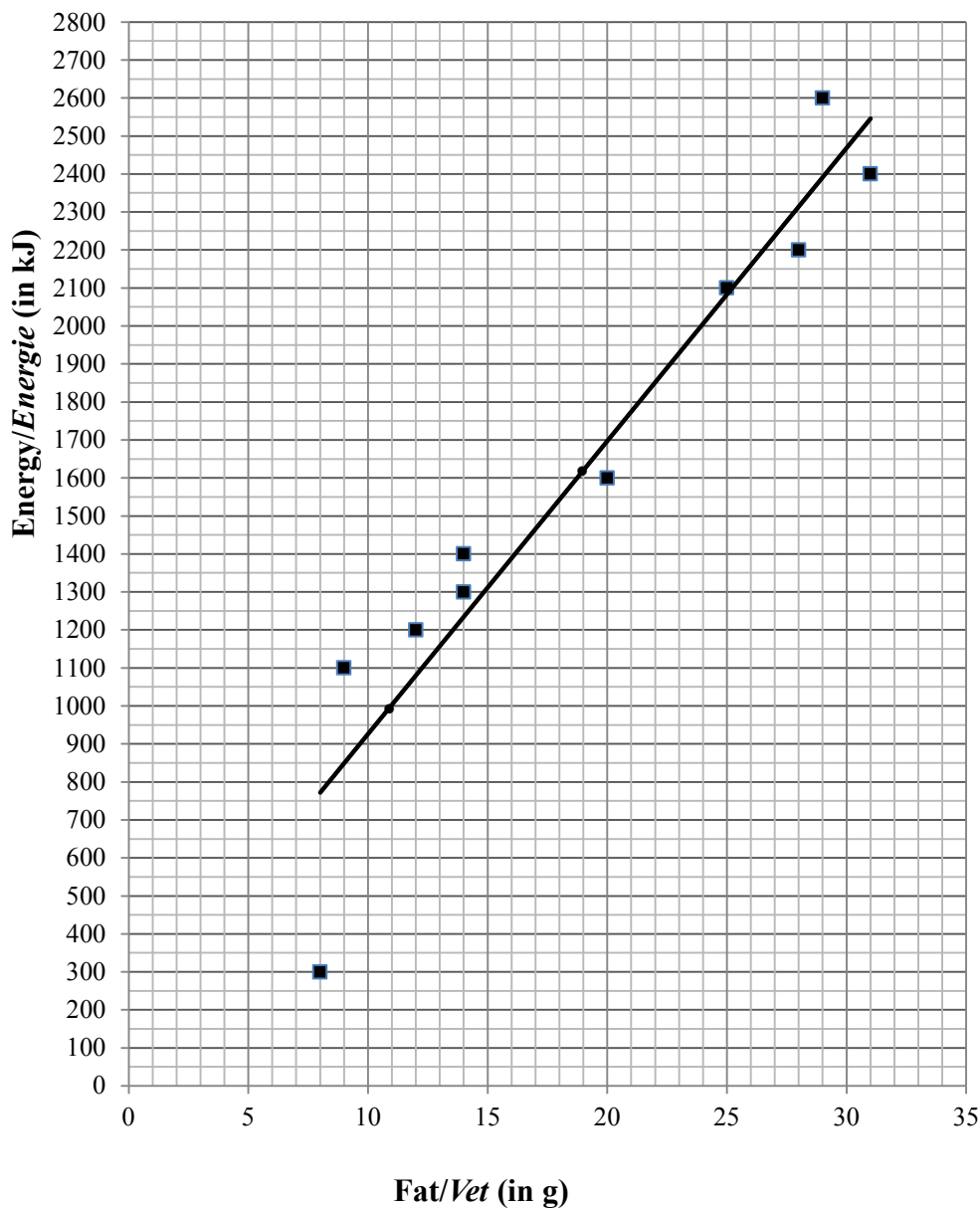
- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek het en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

**QUESTION/VRAAG 1**

<b>Fat/Vet (in g)</b>	9	14	25	8	12	31	28	14	29	20
<b>Energy/Energie (in kJ)</b>	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1

**Scatter plot/Spreidiagram**



1.2.2

1.1  
no marks:  
0 – 2 points correctly  
  
✓ plotting  
3 – 5 points correctly  
  
✓✓ plotting  
6 – 9 points correctly  
  
✓✓✓ plotting  
all 10 points correctly  
  
*geen punte:*  
0 – 2 punte korrek  
  
✓ stip 3 – 5 pte korrek  
  
✓✓ stip 6 – 9 pte korrek  
  
✓✓✓ stip al 10 pte korrek  
  
(3)

1.2.2  
✓ y – int close to (0 ; 150)  
✓ one pt close to (25 ; 2100) or (20 ; 1700)  
(2)

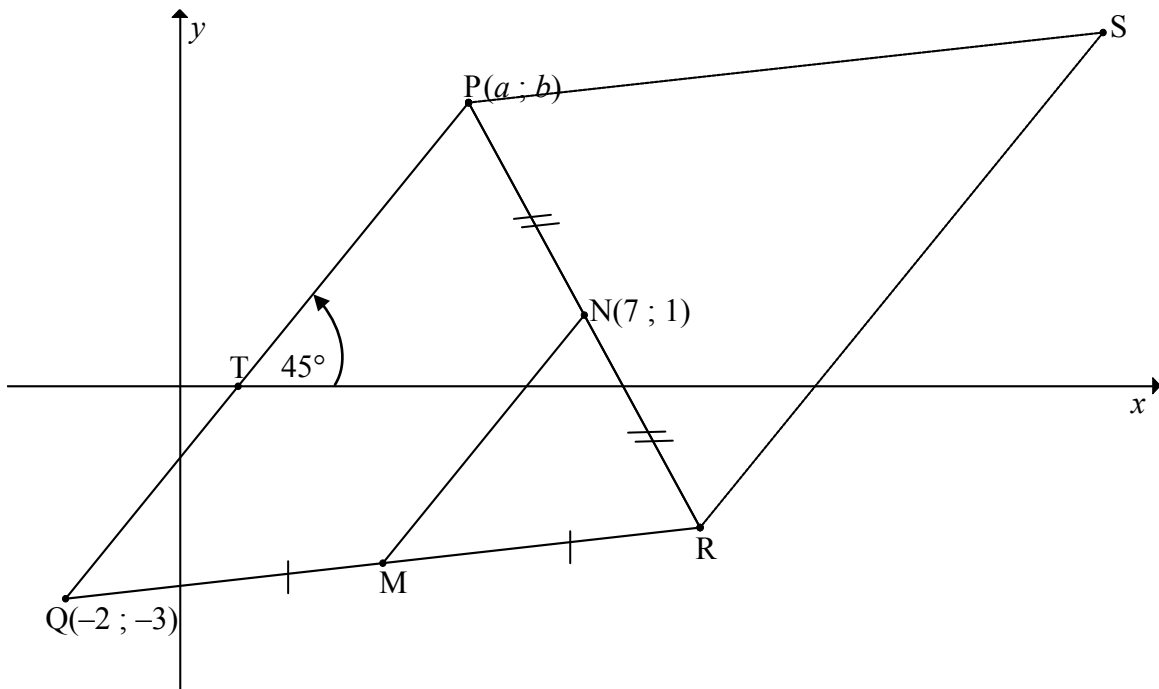
1.2.1	$\hat{y} = 154,60 + 77,13(18)$ $= 1\,542,94 \approx 1\,500 \text{ kJ}$	✓ subst ✓ answ rounded off correctly/ <i>antw korrek</i> <i>afgerond</i> (2)
1.3	(8 ; 300)	✓ answ/ <i>antw</i> (1)
1.4	$r = 0,9520... \approx 0,95$	✓✓ answ/ <i>antw</i> (2)
1.5	very strong positive relationship/ <i>baie sterk positiewe verband</i>	✓ strong/ <i>sterk</i> (1) <b>[11]</b>

**QUESTION/VRAAG 2**

<b>Sum of the values on uppermost faces/ Som van die waardes op boonste vlakke</b>	<b>Frequency/ Frekwensie</b>
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

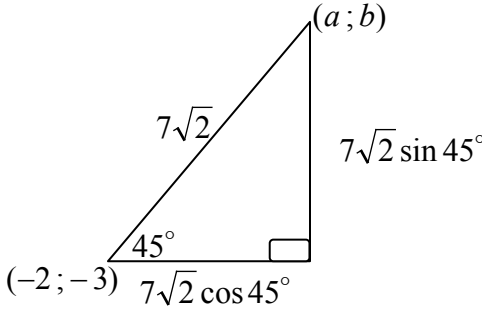
2.1	$\text{mean/gemiddelde} = \frac{2(0) + 3(3) + 4(2) + \dots + 12(1)}{30} = \frac{202}{30}$ $= 6,73$	✓ 202 ✓ answ/antw (2)
2.2	$\text{median/mediaan} = \frac{T_{15} + T_{16}}{2} = \frac{7 + 7}{2} = 7$	✓✓ answ/antw (2)
2.3	$SD/SA = 2,264\dots \approx 2,26$	✓✓ answ/antw (2)
2.4	$(6,73 - 2,26 ; 6,73 + 2,26)$ $= (4,47 ; 8,99)$ $\therefore 4 + 4 + 8 + 3 = 19 \text{ times/keer}$	✓ lower boundary ✓ upper boundary ✓ answ/antw (3) <b>[9]</b>

**QUESTION/VRAAG 3**



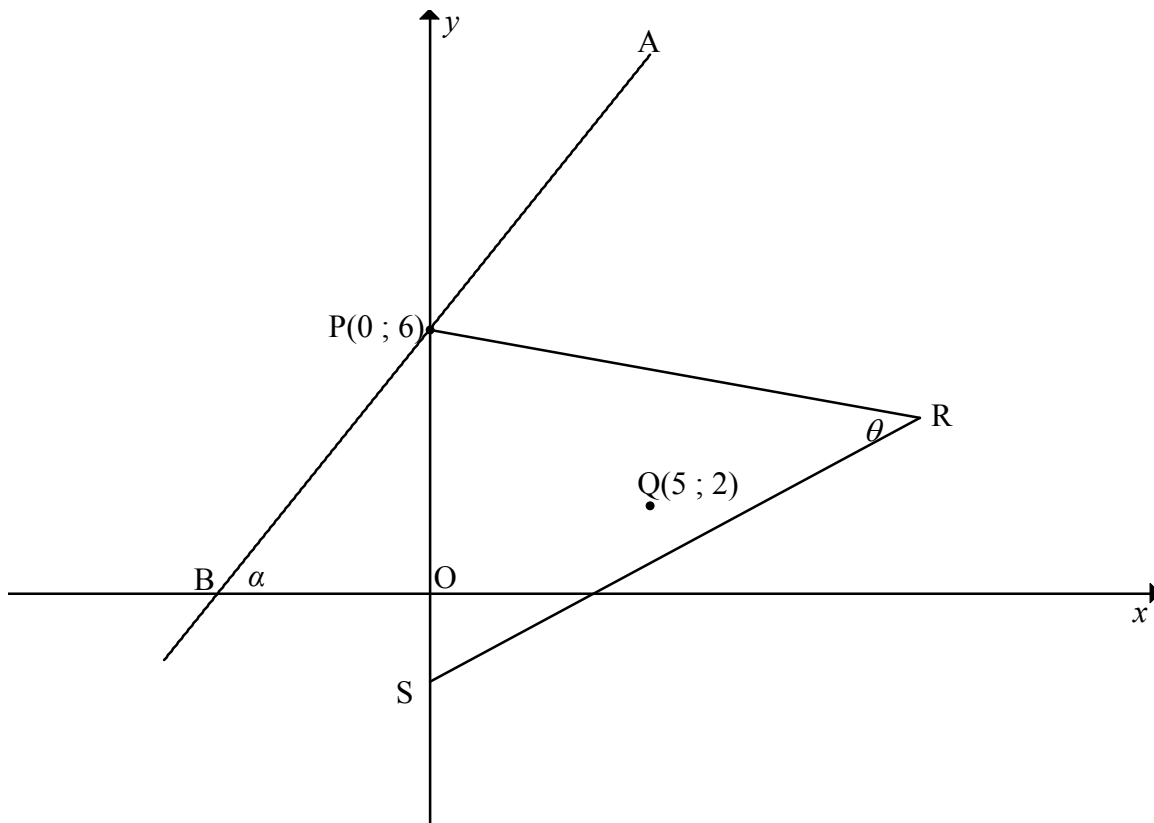
3.1	$m_{PQ} = \tan 45^\circ$ $= 1$	✓ $m = \tan 45^\circ$ ✓ answ/antw (2)
3.2	$MN \parallel QP$ [midpt theorem/midpt-stelling] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore y = x - 6$  <b>OR/OF</b> $MN \parallel PQ$ [midpt theorem/midpt-stelling] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore y = x - 6$	✓ <b>S OR R</b> ✓ $m_{MN}$ ✓ subst $m$ and/en $N(7; 1)$ ✓ equation/vgl (4)  ✓ <b>S OR R</b> ✓ $m_{MN}$ ✓ subst $m$ and/en $N(7; 1)$ ✓ equation/vgl (4)
3.3	$MN = \frac{1}{2} PQ$ [midpoint theorem/midp stelling] $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4,95$	✓ S  ✓ answ/antw (2)

3.5	<p>QN = NS [diag of   m/hoekl van   m]</p> $\frac{-2 + x_s}{2} = 7 \quad \text{and/en} \quad \frac{-3 + y_s}{2} = 1$ <p><math>\therefore x_s = 16</math>                      <math>\therefore y_s = 5</math></p> <p><b>OR/OF</b></p> <p>QN = NS [diag of   m/hoekl van   m]</p> <p><math>\therefore</math> by inspection/deur inspeksie: S(16 ; 5)</p>	<p>✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)</p> <p>✓ method/metode ✓ x-value/waarde ✓ y-value/waarde (3)</p>
3.6	<p>Equation of/Vgl van PQ: <math>y = x + c</math> <math>-3 = -2 + c</math> <math>y = x - 1</math>                      <math>\therefore a = b + 1</math> .....(1)</p> <p>From distance formula/Van afstandformule:</p> $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}$ $\therefore 98 = (a + 2)^2 + (b + 3)^2$ .....(2) <p>Subst (1) into (2):</p> $98 = (b + 1 + 2)^2 + (b + 3)^2$ $98 = b^2 + 6b + 9 + b^2 + 6b + 9$ $0 = 2b^2 + 12b - 80$ $0 = b^2 + 6b - 40$ $\therefore 0 = (b + 10)(b - 4)$ $\therefore b = 4 \quad (\text{since } b > 0)$ <p>Subst <math>b = 4</math> into (1):</p> $\therefore a = 4 + 1 = 5$ $\therefore P(5 ; 4)$ <p><b>OR/OF</b></p> <p>Equation of/Vgl van PQ: <math>y = x + c</math> <math>-3 = -2 + c</math> <math>y = x - 1</math>                      <math>\therefore a = b + 1</math> .....(1)</p> <p>From distance formula/Van afstandformule:</p> $7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}$ $\therefore 98 = (a + 2)^2 + (b + 3)^2$ .....(2) <p>Subst (1) into (2):</p> $98 = (b + 1 + 2)^2 + (b + 3)^2$ $98 = 2(b + 3)^2$ $49 = (b + 3)^2$ $\pm 7 = b + 3$ $\pm 7 - 3 = b$ $\therefore b = 4 \quad (\text{since } b > 0)$ <p>Subst <math>b = 4</math> into (1):</p> $\therefore a = 4 + 1 = 5$ $\therefore P(5 ; 4)$	<p>✓ eq of/vgl van PQ</p> <p>✓ subst Q &amp; <math>7\sqrt{2}</math> into/in distance formula/ afstandformule</p> <p>✓ subst eq of/vgl v. PQ</p> <p>✓ st form/st vorm</p> <p>✓ value of/waarde van b</p> <p>✓ value of/waarde van a (6)</p> <p>✓ eq of/vgl van PQ</p> <p>✓ subst Q &amp; <math>7\sqrt{2}</math> into/in distance formula/ afstandformule</p> <p>✓ subst eq of/vgl v. PQ</p> <p>✓ simplification/ vereenvoudig</p> <p>✓ value of/waarde van b</p> <p>✓ value of/waarde van a (6)</p>

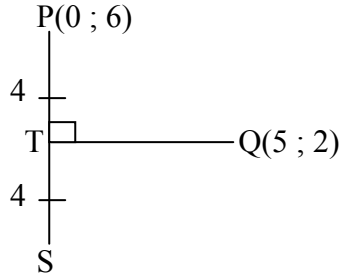
	<p><b>OR/OF</b></p> <p>Equation of/Vgl van PQ: <math>y = x + c</math>  <math>-3 = -2 + c</math>  <math>y = x - 1 \quad \therefore a = b + 1 \quad \dots(1)</math></p> <p>From distance formula/Van afstandformule:  <math>7\sqrt{2} = \sqrt{(a - (-2))^2 + (b - (-3))^2}</math>  <math>98 = (a + 2)^2 + (a - 1 + 3)^2</math>  <math>= 2(a + 2)^2</math>  <math>\therefore a + 2 = 7 \quad (\text{since/aangesien } a &gt; 0)</math>  <math>\therefore a = 5</math>                  Subst <math>a = 4</math> into (1):  <math>\therefore b = 5 - 1 = 4</math>  <math>\therefore P(5 ; 4)</math></p> <p><b>OR/OF</b></p>  <p><math>a = -2 + 7\sqrt{2} \cos 45^\circ = 5</math>  <math>b = -3 + 7\sqrt{2} \sin 45^\circ = 4</math></p>	<p>✓ eq of/vgl van PQ</p> <p>✓ subst Q &amp; <math>7\sqrt{2}</math> into/in distance formula/afstandformule</p> <p>✓ subst eq of/vgl v. PQ</p> <p>✓ simplification/vereenvoudig</p> <p>✓ value of/waarde van a</p> <p>✓ value of/waarde van b</p> <p>(6)</p> <p>✓✓✓✓</p> <p>✓</p> <p>✓</p> <p>(6) [17]</p>
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**QUESTION/VRAAG 4**

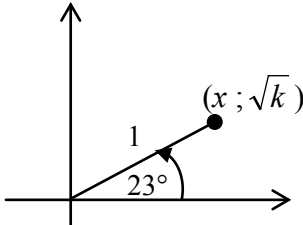


<p>4.1</p>	$(x - 5)^2 + (y - 2)^2 = r^2$ $(0 - 5)^2 + (6 - 2)^2 = r^2$ $25 + 16 = r^2$ $41 = r^2$ $\therefore (x - 5)^2 + (y - 2)^2 = 41$ <p><b>OR/OF</b></p> $PQ = \sqrt{(0 - 5)^2 + (6 - 2)^2}$ $= \sqrt{25 + 16}$ $r = \sqrt{41}$ $\therefore (x - 5)^2 + (y - 2)^2 = 41$	<ul style="list-style-type: none"> <li>✓ subst (5 ; 2) into circle eq/in sirkelvgl</li> <li>✓ value of/waarde van <math>r^2</math></li> <li>✓ equation/vgl (3)</li> <li>✓ subst (5 ; 2) &amp; (0 ; 6) into dist. form/in afst. form</li> <li>✓ value of/waarde van <math>r</math></li> <li>✓ equation/vgl (3)</li> </ul>
<p>4.2</p>	$(0 - 5)^2 + (y - 2)^2 = 41$ $25 + (y - 2)^2 = 41$ $25 + y^2 - 4y + 4 = 41$ $y^2 - 4y - 12 = 0$ $(y - 6)(y + 2) = 0$ $y \neq 6 \text{ or / of } y = -2$ $\therefore S(0 ; -2) \text{ or } y = -2$	<ul style="list-style-type: none"> <li>✓ <math>x = 0</math></li> <li>✓ st form/st. vorm</li> <li>✓ answ/antw (neg value) (3)</li> </ul>

	<p><b>OR/OF</b></p> $(0 - 5)^2 + (y - 2)^2 = 41$ $25 + (y - 2)^2 = 41$ $(y - 2)^2 = 16$ $y - 2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6 \quad \text{or / of} \quad y = -2$ <p><math>\therefore S(0 ; -2)</math></p> <p><b>OR/OF</b></p> <p>Draw/Trek QT <math>\perp</math> PS  PT = TS [line from centre <math>\perp</math> to chord/  lyn van midpt <math>\perp</math> koord]</p> $PT = y_P - y_Q = 6 - 2 = 4$ $y_Q - y_S = 4$ $y_S = 2 - 4 = -2$ <p><math>\therefore S(0 ; -2)</math></p> 	<p><math>\checkmark x = 0</math></p> <p><math>\checkmark</math> square form/ kwadraatvorm</p> <p><math>\checkmark</math> answ/antw (neg value)</p> <p>(3)</p> <p><math>\checkmark x = 0</math></p> <p><math>\checkmark\checkmark y = -2</math></p> <p>(3)</p>
<p>4.3</p>	$m_{PQ} = \frac{6 - 2}{0 - 5}$ $= -\frac{4}{5}$ <p><math>m_{PQ} \times m_{APB} = -1</math> [tan/raakl <math>\perp</math> radius]</p> $\therefore m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	<p><math>\checkmark</math> subst (0 ; 6) &amp; (5 ; 2) into grad form/in grad. formule</p> <p><math>\checkmark m_{PQ}</math></p> <p><math>\checkmark m_{APB}</math></p> <p><math>\checkmark</math> equation/vgl</p> <p>(4)</p>
<p>4.4</p>	$\tan \alpha = \frac{5}{4}$ $\therefore \alpha = 51,34^\circ$ <p><b>OR/OF</b></p> <p>B(4,8 ; 0)</p> $\therefore \tan \alpha = \frac{6}{4,8}$ $\therefore \alpha = 51,34^\circ$	<p><math>\checkmark \tan \alpha = m_{APB}</math></p> <p><math>\checkmark</math> answ/antw</p> <p>(2)</p> <p><math>\checkmark \tan \alpha = \frac{6}{4,8}</math></p> <p><math>\checkmark</math> answ/antw</p> <p>(2)</p>

<p>4.5</p>	<p> <math>\theta = \hat{B}\hat{P}\hat{S}</math> [tan-chord th/raakl-koordst.]  <math>= 90^\circ - \alpha</math> [<math>\angle</math> sum in <math>\Delta/\angle</math> som van <math>\Delta</math>]  <math>= 90^\circ - 51,34^\circ</math>  <math>= 38,66^\circ</math> </p> <p><b>OR/OF</b></p> <p> <math>PS = 8</math>  <math>PQ = SQ = \sqrt{41}</math>  <math>PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos\hat{P}\hat{Q}\hat{S}</math>  <math>64 = 41 + 41 - 2.41.\cos\hat{P}\hat{Q}\hat{S}</math>  <math>\cos\hat{P}\hat{Q}\hat{S} = \frac{18}{82}</math>  <math>\hat{P}\hat{Q}\hat{S} = 77,32^\circ</math>  <math>\theta = \frac{1}{2}\hat{P}\hat{Q}\hat{S}</math> [<math>\angle</math> at centre = <math>2 \times \angle</math> circumf]  <math>= 38,66^\circ</math> </p>	<p> <math>\checkmark</math> S <math>\checkmark</math> R  <math>\checkmark</math> <math>90^\circ - \alpha</math>  <math>\checkmark</math> answ/antw (4)                 </p> <p> <math>\checkmark</math> correct subst into cosine rule  <math>\checkmark</math> <math>\hat{P}\hat{Q}\hat{S} = 77,32^\circ</math>  <math>\checkmark</math> R  <math>\checkmark</math> answ/antw (4)                 </p>
<p>4.6</p>	<p> <math>\text{Area } \Delta PQS = \frac{1}{2} PS \times \text{height/hoogte}</math>  <math>= \frac{1}{2} (8)(5)</math>  <math>= 20 \text{ sq units/vk eenh}</math> </p> <p><b>OR/OF</b></p> <p> <math>\hat{P}\hat{Q}\hat{S} = 2 \times 38,66^\circ</math> [<math>\angle</math> at centre = <math>2 \times \angle</math> at circum/  <i>midpts <math>\angle = 2</math> omtreks <math>\angle</math></i>]  <math>= 77,32^\circ</math>  <math>\text{Area } \Delta PQS = \frac{1}{2} PQ.QS.\sin\hat{P}\hat{Q}\hat{S}</math>  <math>= \frac{1}{2} \cdot \sqrt{41} \cdot \sqrt{41} \cdot \sin 77,32^\circ</math>  <math>= 20 \text{ sq units/vk eenh}</math> </p>	<p> <math>\checkmark</math> area formula/e:  <math>\Delta PQS</math>  <math>\checkmark</math> <math>PS = 8</math>  <math>\checkmark</math> <math>\perp h = 5</math>  <math>\checkmark</math> answ/antw (4)                 </p> <p> <math>\checkmark</math> size of/grootte v <math>\hat{P}\hat{Q}\hat{S}</math>  <math>\checkmark</math> area rule/reël:  <math>\Delta PQS</math>  <math>\checkmark</math> subst correctly/  <i>subst korrek</i>  <math>\checkmark</math> answ/antw (4)                 </p> <p style="text-align: right;"><b>[20]</b></p>

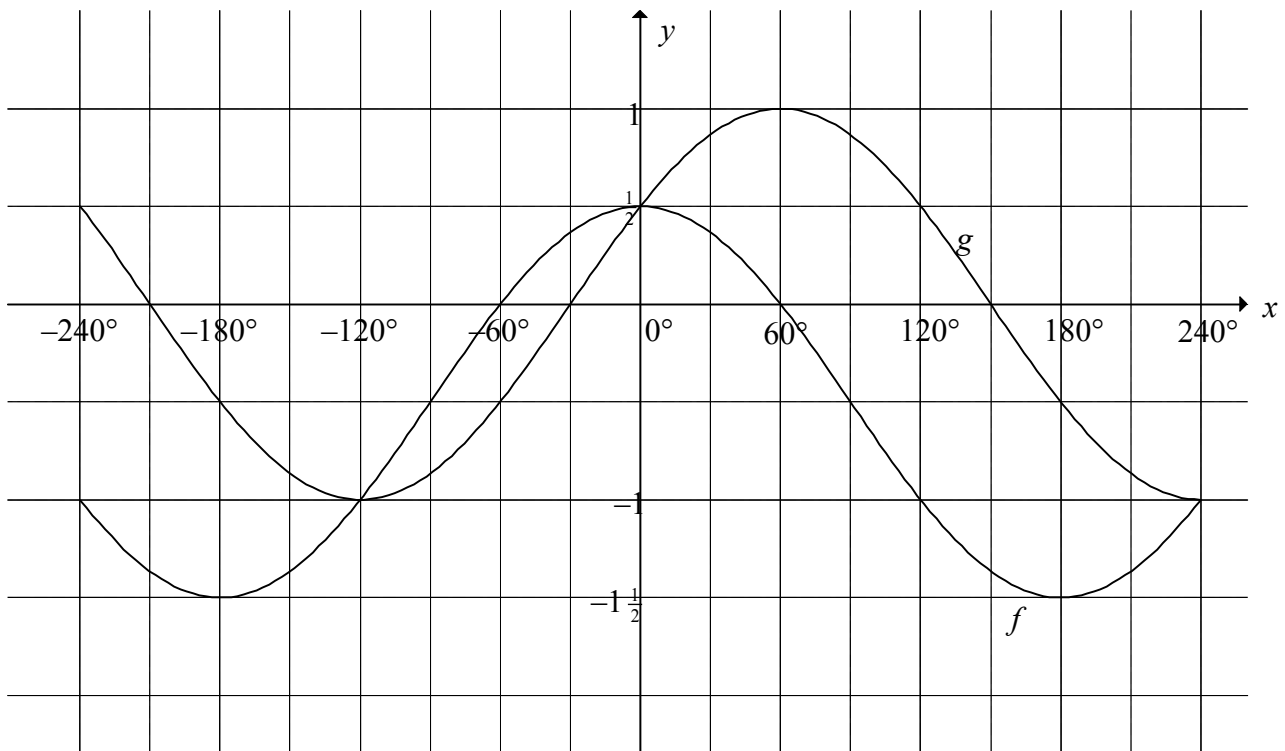
**QUESTION/VRAAG 5**

5.1.1	$\sin 203^\circ$ $= -\sin 23^\circ$ $= -\sqrt{k}$	✓ reduction/ <i>reduksie</i> ✓ answ ito/antw itv k (2)
5.1.2	$\cos^2 23^\circ = 1 - \sin^2 23^\circ$ $= 1 - k$ $\cos 23^\circ = \sqrt{1 - k}$ <p><b>OR/OF</b></p> $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1 - k}$ $\cos 23^\circ = \frac{\sqrt{1 - k}}{1} = \sqrt{1 - k}$ 	✓ identity/identiteit ✓ $\cos^2 23^\circ$ ito/itv k ✓ answ/antw (3) ✓ $x^2 = 1 - k$ ✓ x ito/itv k ✓ answ/antw (3)
5.1.3	$\tan (-23^\circ) = -\tan 23^\circ$ $= -\frac{\sin 23^\circ}{\cos 23^\circ}$ $= -\frac{\sqrt{k}}{\sqrt{1 - k}} = -\sqrt{\frac{k}{1 - k}}$ <p><b>OR/OF</b></p> $\tan (-23^\circ) = -\tan 23^\circ$ $= -\frac{\sqrt{k}}{\sqrt{1 - k}} = -\sqrt{\frac{k}{1 - k}}$	✓ reduction/ <i>reduksie</i> ✓ answ ito/antw itv k (2) ✓ reduction/ <i>reduksie</i> ✓ answ ito/antw itv k (2)
5.2	$\frac{4 \cos x \cdot (-\sin x)}{\sin(30^\circ - x + x)}$ $= \frac{-4 \sin x \cdot \cos x}{\sin 30^\circ}$ $= \frac{-4 \sin x \cdot \cos x}{\frac{1}{2}}$ $= -8 \sin x \cdot \cos x$ $= -4(2 \sin x \cdot \cos x)$ $= -4 \sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\frac{1}{2}$ ✓ double sine form / <i>dubbel sin form</i> ✓ answ/antw (6)

	<p><b>OR/OF</b></p> $\frac{4 \cos x \cdot (-\sin x)}{(\sin 30^\circ \cos x - \cos 30^\circ \sin x) \cos x + (\cos 30^\circ \cos x + \sin 30^\circ \sin x) \sin x}$ $= \frac{-4 \sin x \cdot \cos x}{\left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x\right) \cos x + \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x\right) \sin x}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} (\cos^2 x + \sin^2 x)}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} (1)}$ $= -8 \cos x \sin x$ $= -4(2 \sin x \cos x)$ $= -4 \sin 2x$	<p>✓ <math>\cos x</math> ✓ <math>-\sin x</math></p> <p>✓</p> <p><math>\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x</math></p> <p>✓ <math>\frac{1}{2}</math></p> <p>✓ double sine form / <i>dubbel sin form</i></p> <p>✓ <i>answ/antw</i></p> <p>(6)</p>
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5.3	$\cos 2x - 7 \cos x - 3 = 0$ $2 \cos^2 x - 1 - 7 \cos x - 3 = 0$ $2 \cos^2 x - 7 \cos x - 4 = 0$ $(2 \cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2} \text{ or/of } \cos x = 4 \text{ (no solution)}$ $\therefore x = 120^\circ + n.360^\circ \text{ or/of } x = 240^\circ + n.360^\circ ; n \in \mathbb{Z}$ <p><b>OR/OF</b></p> $\therefore x = \pm 120^\circ + n.360^\circ ; n \in \mathbb{Z}$	<ul style="list-style-type: none"> <li>✓ expansion/ uitbreiding</li> <li>✓</li> <li><math>2 \cos^2 x - 7 \cos x - 4 = 0</math></li> <li>✓ factors/faktore</li> <li>✓ <math>\cos x = -\frac{1}{2}</math></li> <li>✓ <math>120^\circ</math> &amp; <math>240^\circ</math></li> <li>✓ <math>+ n.360^\circ</math></li> <li><b>OR/OF</b></li> <li>✓ <math>\pm 120^\circ</math></li> <li>✓ <math>+ n.360^\circ</math></li> </ul> <p style="text-align: right;">(6)</p>
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ $= 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	<ul style="list-style-type: none"> <li>✓ expansion of/ uitbreiding van <math>\sin(2\theta + \theta)</math></li> <li>✓ expansions of <math>\sin 2\theta</math> AND <math>\cos 2\theta</math></li> <li>✓ <math>1 - \sin^2 \theta</math></li> <li>✓ subst</li>   <li>✓ answ/antw</li> </ul> <p style="text-align: right;">(5) <b>[24]</b></p>

**QUESTION/VRAAG 6**

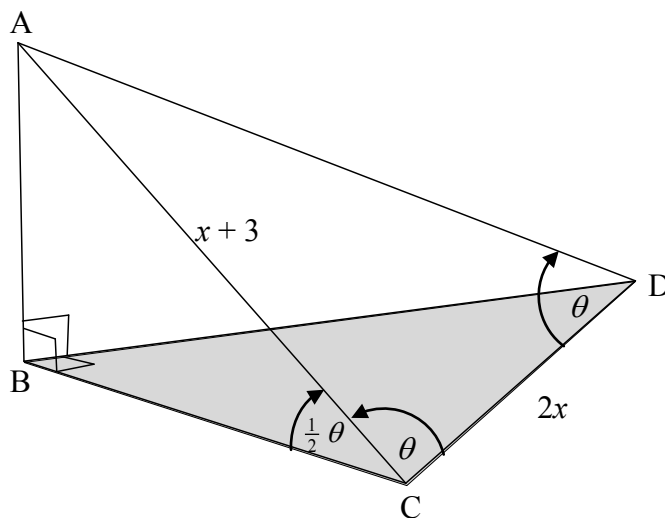


<p>6.1</p>	<p><math>f(x) = \cos x - \frac{1}{2}</math> and/en <math>g(x) = \sin(x + 30^\circ)</math>  <math>\therefore p = 30^\circ</math> and/en <math>q = -\frac{1}{2}</math></p> <p><b>OR/OF</b></p> <p><math>\sin(60^\circ + p) = 1</math> and/en <math>\cos 0^\circ + q = \frac{1}{2}</math>  <math>\therefore p = 30^\circ</math> and/en <math>\therefore q = -\frac{1}{2}</math></p>	<p>✓ <math>f(x) = \cos x - \frac{1}{2}</math>                  ✓ <math>g(x) = \sin(x + 30^\circ)</math>                  ✓ value of/waarde v p                  ✓ value of/waarde v q                  (4)</p> <p>✓ <math>\sin(60^\circ + p) = 1</math>                  ✓ <math>\cos 0^\circ + q = \frac{1}{2}</math>                  ✓ value of/waarde v p                  ✓ value of/waarde v q                  (4)</p>
<p>6.2</p>	<p><math>x \in (-120^\circ ; 0^\circ)</math> <b>OR/OF</b> <math>-120^\circ &lt; x &lt; 0^\circ</math></p>	<p>✓ critical values/ kritiese waardes                  ✓ correct interval/ korrekte interval                  (2)</p>

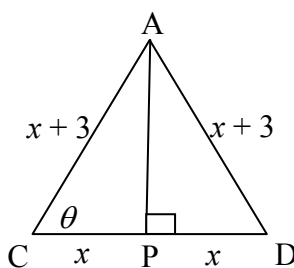
6.3	<p>The graph of <math>g</math> has to shift <math>60^\circ</math> to the left and then be reflected about the <math>x</math>-axis./<i>Die grafiek van <math>g</math> moet <math>60^\circ</math> na links skuif en dan om die <math>x</math>-as gereflekteer word.</i></p> <p><b>OR/OF</b></p> <p>The graph of <math>g</math> must be reflected about the <math>x</math>-axis and then be shifted <math>60^\circ</math> to the left./<i>Die grafiek van <math>g</math> moet om die <math>x</math>-as gereflekteer word en dan met <math>60^\circ</math> na links geskuif word.</i></p> <p><b>OR/OF</b></p> <p>The graph of <math>g</math> has to shift <math>120^\circ</math> to the right./<i>Die grafiek van <math>g</math> moet <math>120^\circ</math> na regs geskuif word.</i></p> <p><b>OR/OF</b></p> <p>The graph of <math>g</math> has to shift <math>240^\circ</math> to the left./<i>Die grafiek van <math>g</math> moet met <math>240^\circ</math> na links geskuif word</i></p>	<p>✓ <math>60^\circ</math> left/<i>links</i>  ✓ reflection about <math>x</math>-axis/<i>refleksie om <math>x</math>-as</i>  (2)</p> <p>✓ reflection about <math>x</math>-axis/<i>refleksie om <math>x</math>-as</i>  ✓ <math>60^\circ</math> left/<i>links</i>  (2)</p> <p>✓ ✓ <math>120^\circ</math> right/<i>regs</i>  (2)</p> <p>✓ ✓ <math>240^\circ</math> left/<i>links</i>  (2)</p> <p><b>[8]</b></p>
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**QUESTION/VRAAG 7**

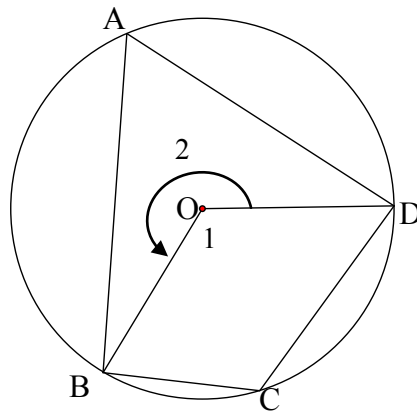


7.1	$\hat{C}AD = 180^\circ - 2\theta$ [ $\angle$ s sum of $\Delta$ / $\angle$ e som van $\Delta$ ]	✓ answ/antw (1)
7.2	$\frac{\sin \theta}{x+3} = \frac{\sin(180^\circ - 2\theta)}{2x}$ $\frac{\sin \theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin \theta}{x+3} = \frac{2 \sin \theta \cdot \cos \theta}{2x}$ $\cos \theta = \frac{2x \sin \theta}{2(x+3) \sin \theta}$ $\cos \theta = \frac{x}{x+3}$ <p><b>OR/OF</b>  <math>AD = x + 3</math> [sides opp = <math>\angle</math>s/sye to = <math>\angle</math>e]  <math>AC^2 = AD^2 + CD^2 - 2AD \cdot CD \cdot \cos \theta</math>  <math>(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3) \cdot \cos \theta</math>  <math>0 = 4x^2 - 4x(x+3) \cos \theta</math>  <math>\cos \theta = \frac{4x^2}{4x(x+3)}</math>  <math>= \frac{x}{x+3}</math></p> <p><b>OR/OF</b>                  Draw/Trek <math>AP \perp CD</math></p> $\cos \theta = \frac{x}{x+3}$	✓ correct subst into sine rule/korrekte subst in sin-reël ✓ $\sin 2\theta$ ✓ $2 \sin \theta \cdot \cos \theta$ ✓ $\cos \theta$ as subject/as onderwerp (4) ✓ $AD = x + 3$ ✓ correct subst into cosine rule/korrekte subst in cos-reël ✓ simplification/vereenvoudiging ✓ $\cos \theta$ as subject/as onderwerp (4) ✓ ✓ constr/konstr ✓ ✓ sketch shown/toon skets (4)



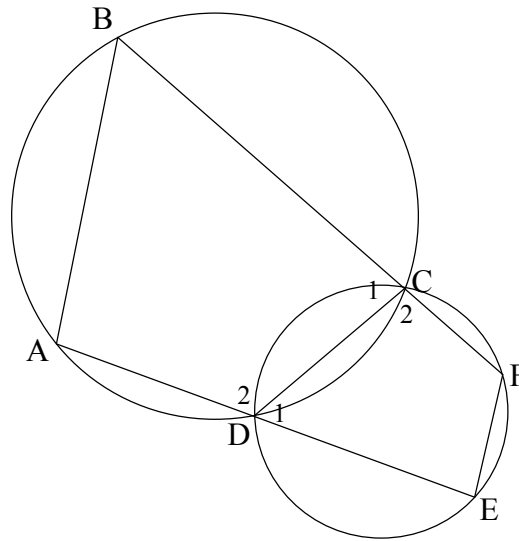
7.3	$\cos \theta = \frac{2}{5}$ $\therefore \theta = 66,42^\circ$ <p>In <math>\triangle ABC</math>:</p> $\sin \frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33,21^\circ = \frac{AB}{5}$ $\therefore AB = 5 \sin 33,21^\circ$ $= 2,74$ <p><b>OR/OF</b></p> $\sin \frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5 \sin \frac{\theta}{2}$ <p>but/maar:</p> $\cos \theta = \frac{2}{5}$ $1 - 2 \sin^2 \frac{\theta}{2} = \frac{2}{5}$ $\sin^2 \frac{\theta}{2} = \frac{3}{10}$ $\sin \frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5 \sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2,74$	$\checkmark \cos \theta = \frac{2}{5}$ $\checkmark \text{ size of/grootte v } \theta$ $\checkmark \text{ correct ratio/}$ $\text{korrekte verh}$ $\checkmark \text{ subst correctly/}$ $\text{korrek}$ $\checkmark \text{ answ/antw}$ <p style="text-align: right;">(5)</p> $\checkmark AB = 5 \sin \frac{\theta}{2}$ $\checkmark \text{ equation/vgl}$ $\checkmark \text{ simplification/}$ $\text{vereenvoudiging}$ $\checkmark \text{ value of/waarde v}$ $\sin \frac{\theta}{2}$ $\checkmark \text{ answ/antw}$ <p style="text-align: right;">(5) <b>[10]</b></p>
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**QUESTION/VRAAG 8**



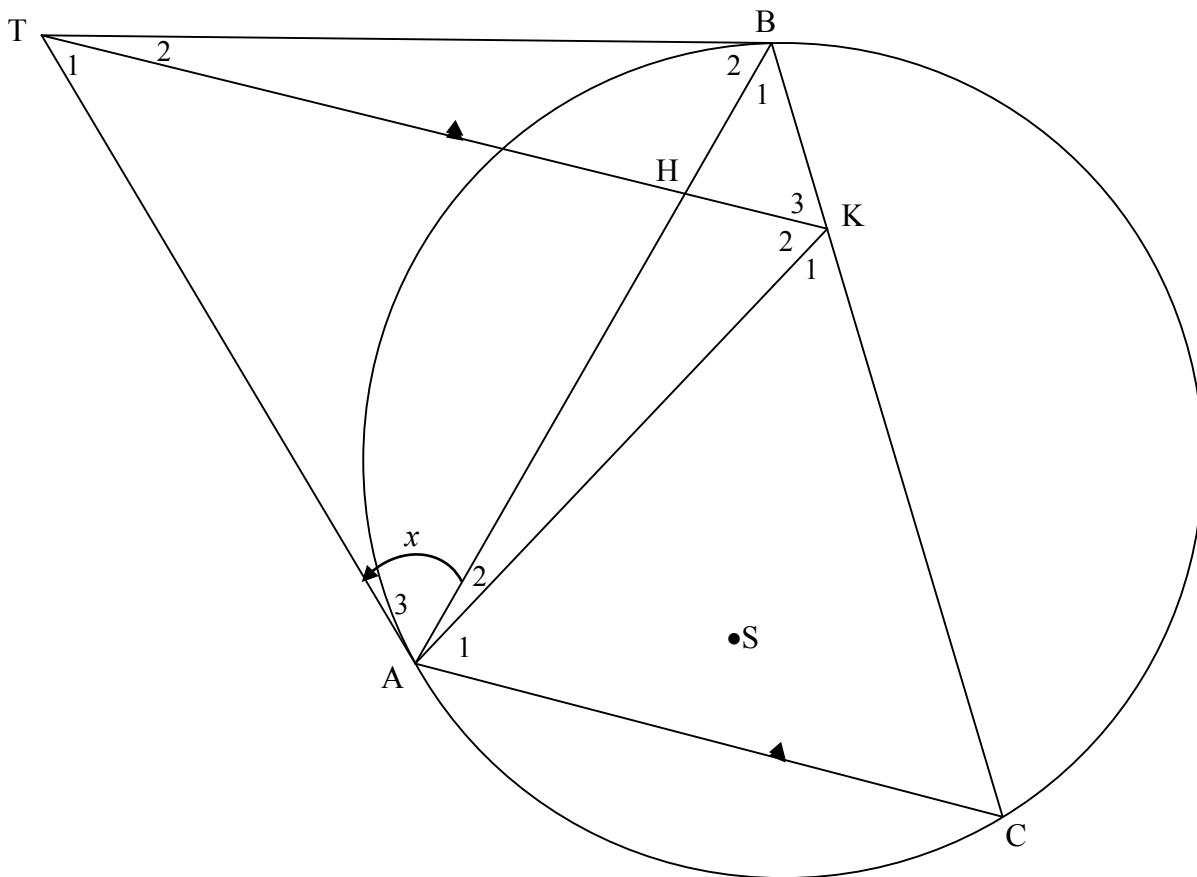
8.1.1	twice or double /twee keer of dubbel	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [∠s in a rev/∠e in omw of om 'n pt] $2\hat{A} + 2\hat{C} = 360^\circ$ $\therefore \hat{A} + \hat{C} = 180^\circ$  <b>OR/OF</b>  Let/Gestel $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\hat{O}_2 = 360^\circ - 2x$ [∠s in a rev/∠e in omw of om 'n pt] $\hat{C} = 180^\circ - x$ [∠ at centre = 2×∠ at circ/midpts ∠ = 2×omtreks ∠] $\therefore \hat{A} + \hat{C} = 180^\circ$	✓ S  ✓ S ✓ S   ✓ S ✓ S ✓ S   (3)

8.2



8.2	$\hat{A} = \hat{C}_2$ $\hat{E} = 180^\circ - \hat{C}_2$ $\therefore \hat{E} = 180^\circ - \hat{A}$ $\therefore EF \parallel AB$  <b>OR/OF</b> $\hat{B} = \hat{D}_1$ $\hat{F} = 180^\circ - \hat{D}_1$ $\therefore \hat{F} = 180^\circ - \hat{B}$ $\therefore EF \parallel AB$	[ext $\angle$ of cyclic quad/ <i>buite</i> $\angle$ v <i>kdvh</i> ] [opp $\angle$ s of cyclic quad/ <i>tos</i> $\angle$ e v <i>kdvh</i> ]  [co-interior $\angle$ s $180^\circ$ / <i>ko-binne</i> $\angle$ e $180^\circ$ ]  [ext $\angle$ of cyclic quad/ <i>buite</i> $\angle$ v <i>kdvh</i> ] [opp $\angle$ s of cyclic quad/ <i>tos</i> $\angle$ e v <i>kdvh</i> ]  [co-interior $\angle$ s $180^\circ$ / <i>ko-binne</i> $\angle$ e $180^\circ$ ]	✓ S ✓ R ✓ S ✓ R  ✓ R (5)  ✓ S ✓ R ✓ S ✓ R  ✓ R (5) <b>[9]</b>
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**QUESTION/VRAAG 9**

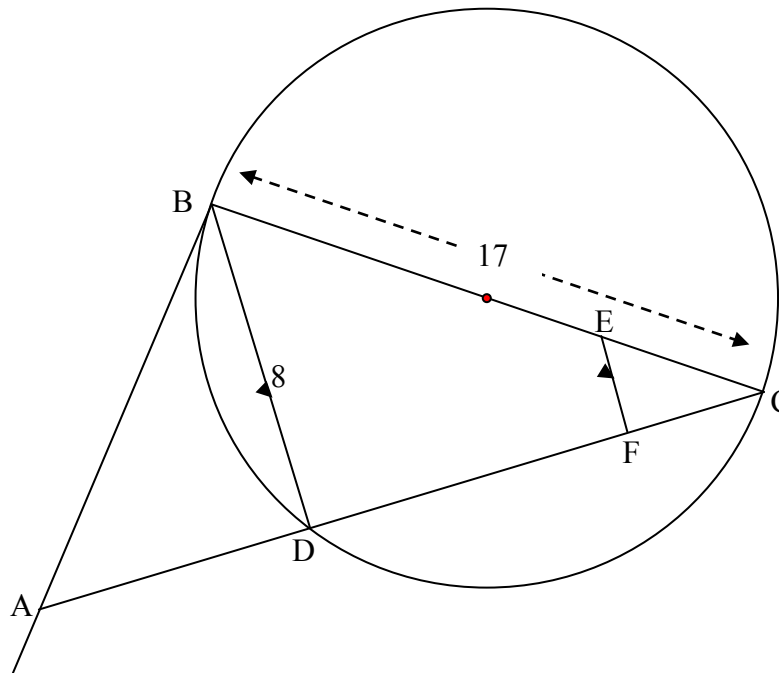


9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp $\angle$ s/ooreenk $\angle e$ ; CA   KT] [tan-chord th/raakl-koordst]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ $\therefore$ AKBT is cyc quad	[proved/bewys in 9.1] [line (BT) subtends equal $\angle$ s/ lyn (BT) onderspan gelyke $\angle e$ ] <b>OR/OF</b> [converse $\angle$ s in same segment/ omgek $\angle e$ in dies segment]	$\checkmark$ S $\checkmark$ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ $\therefore$ TK bisects/halveer $\hat{A}\hat{K}\hat{B}$ <b>OR/OF</b> $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/raakl-koordst] [ $\angle$ s in the same segm/ $\angle e$ in dies segm] [ $\angle$ s in the same seg/ $\angle e$ in dies segm] [tans from same pt; $\angle$ s opp equal sides/ rkle v dies pt; $\angle e$ to gelyke sye]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)

	$\therefore = \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/halveer $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/bewys] $\therefore$ TA tangent [converse tan chord theorem OR $\angle$ between line and chord/ <i>omgekeerde raakl-kdst OF <math>\angle</math> tussen lyn en koord</i> ]	✓ S ✓ R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/konsiklies] $A\hat{T}B = 180^\circ - 2x$ [A,T,B & K concyclic/konsiklies] $\therefore$ points A, S, B and T are also concyclic/ <i>punte A, S, B en T is ook konsiklies</i> [opp $\angle$ s of quad = $180^\circ$ /tos $\angle$ e van vierhoek= $180^\circ$ ]  <b>OR/OF</b>  A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic.  <b>OR/OF</b>  The circle passing through points A, K and B contains the point S on the circumference (A, S, K and B concyclic)./ <i>Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).</i> The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./ <i>Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).</i> $\therefore$ points A, S, B and T are also concyclic/ <i>punte A, S, B en T is konsiklies</i>	✓ S (both/beide statements/bewerings) ✓ R (2)  ✓ S ✓ S (2)  ✓ S ✓ S (2)

[14]

**QUESTION/VRAAG 10**



<p>10.1</p>	<p><math>\hat{BDC} = 90^\circ</math>  <math>DC^2 = 17^2 - 8^2</math>  <math>= 225</math>  <math>\therefore DC = 15</math></p>	<p>[<math>\angle</math> in semi circle/<math>\angle</math> in halfsirkel]                  [Th of/stelling v Pythagoras]</p>	<p>✓ S                  ✓ using/gebruik                  Pyth korrek/                  correctly                  ✓ answ/antw                  (3)</p>
<p>10.2.1</p>	<p><math>\frac{CF}{CD} = \frac{CE}{CB}</math>  <math>\therefore \frac{CF}{15} = \frac{1}{4}</math>  <math>\therefore CF = 3,75</math></p>	<p>[line    one side of <math>\Delta</math>/lyn    een sy van <math>\Delta</math>]  <b>OR/OF</b> <math>\Delta CEF \sim \Delta CBD</math></p>	<p>✓ S/R                  ✓ subst correctly/                  korrek                  ✓ answ/antw                  (3)</p>
<p>10.2.2</p>	<p><math>\hat{BDC} = 90^\circ</math>  <math>\hat{EFC} = \hat{BDC}</math>  <math>\hat{ABC} = 90^\circ</math>                  In <math>\Delta BAC</math> and/en <math>\Delta FEC</math>:  <math>\hat{ABC} = \hat{EFC}</math> [proven/bewys]  <math>\hat{C} = \hat{C}</math> [common/gemeen]  <math>\therefore \Delta BAC \sim \Delta FEC</math> [<math>\angle\angle\angle</math>]  <b>OR/OF</b>  <math>\hat{BDC} = 90^\circ</math>  <math>\hat{EFC} = \hat{BDC}</math>  <math>\hat{ABC} = 90^\circ</math>                  In <math>\Delta BAC</math> and/en <math>\Delta FEC</math>:  <math>\hat{ABC} = \hat{EFC}</math> [proven/bewys]  <math>\hat{C} = \hat{C}</math> [common/gemeen]</p>	<p>[<math>\angle</math> in semi circle/<math>\angle</math> in halfsirkel]                  [corresp <math>\angle</math>s/ooreenk <math>\angle</math>e; EF   BD]                  [tan <math>\perp</math> diameter/raakl <math>\perp</math> middellyn]                  [proven/bewys]                  [common/gemeen]                  [<math>\angle\angle\angle</math>]                  [<math>\angle</math> in semi circle/<math>\angle</math> in halfsirkel]                  [corresp <math>\angle</math>s/ooreenk <math>\angle</math>e; EF   BD]                  [tan <math>\perp</math> diameter/raakl <math>\perp</math> middellyn]                  [proven/bewys]                  [common/gemeen]</p>	<p>✓ S/R                  ✓ S ✓ R                  ✓ S                  ✓ R                  (5)                  ✓ S/R                  ✓ S ✓ R                  ✓ S</p>

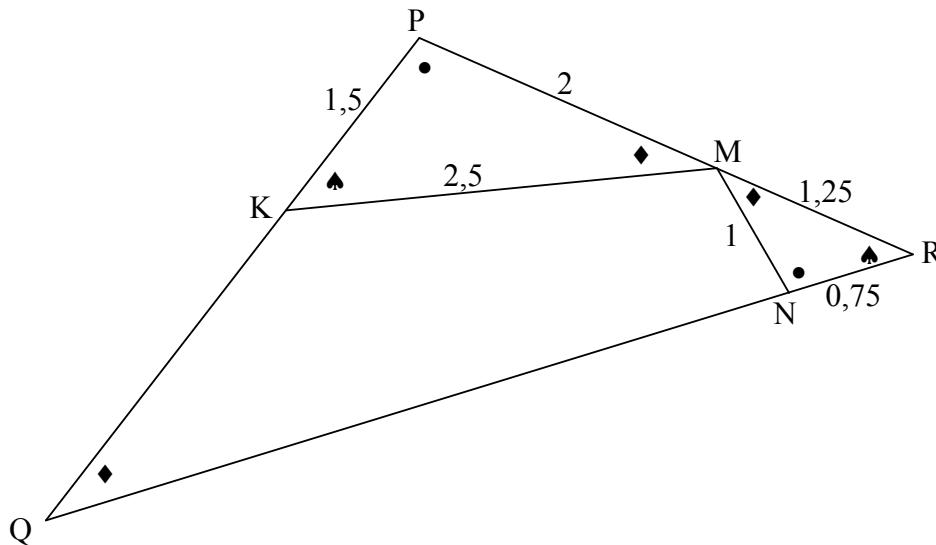
	$\hat{B}AC = \hat{F}EC \quad [\angle \text{sum in } \Delta / \angle \text{ som van } \Delta]$ $\therefore \Delta BAC \parallel \Delta FEC$	<p>✓ S</p> <p>(5)</p>
<p>10.2.3</p>	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC} \quad [\Delta BAC \parallel \Delta FEC]$ $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27 \text{ or/of } 19\frac{4}{15}$ <p><b>OR/OF</b></p> $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27 \text{ or/of } 19\frac{4}{15}$ <p><b>OR/OF</b></p> $\Delta BCA \parallel \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27 \text{ or/of } 19\frac{4}{15}$ <p><b>OR/OF</b></p> $\hat{C} = \hat{A}BD \quad [\text{tan-chord theorem/rkl-kdstelling}]$ $\frac{AD}{8} = \tan \hat{A}BD$ $= \tan \hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27 \text{ or/of } 19\frac{4}{15}$	<p>✓ length of/lengte v EC</p> <p>✓ S</p> <p>✓ subst correctly/ korrek</p> <p>✓ answ/antw</p> <p>(4)</p> <p>✓ ✓ correct ratios/ korrekte verh's</p> <p>✓ subst correctly/ korrek</p> <p>✓ answ/antw</p> <p>(4)</p> <p>✓ S <b>OR</b> Pyth th</p> <p>✓ correct ratio</p> <p>✓ subst</p> <p>✓ answ/antw</p> <p>(4)</p> <p>✓ S</p> <p>✓ correct ratio</p> <p>✓ subst</p> <p>✓ answ/antw</p> <p>(4)</p>



10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends $90^\circ$ <b>OR</b> converse $\angle$ in semi circle ] <i>AC is middellyn van die sirkel wat deur die punte A, B en C gaan</i> [koord onderspan $90^\circ$ <b>OF</b> omgek $\angle$ in halfsirkel ] $\therefore \text{radius} = \frac{1}{2} \times 19,27 = 9,63$ or/of $9\frac{19}{30}$ or/of $\frac{1}{2} AC$	✓ S/R  ✓ answ/antw  (2) <b>[17]</b>
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**QUESTION/VRAAG 11**

11.1	equiangular or similar/ <i>gelykhoekig of gelykvormig</i>	✓ answ/antw (1)
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11.2.1	$\frac{KP}{RN} = \frac{1,5}{0,75} = 2 ; \frac{PM}{NM} = \frac{2}{1} = 2 ; \frac{KM}{RM} = \frac{2,5}{1,25} = 2$ $\therefore \frac{KP}{RN} = \frac{PM}{NM} = \frac{KM}{RM}$ <p><math>\therefore \Delta KPM \parallel \Delta RNM</math> [Sides of <math>\Delta</math> in prop/sye v <math>\Delta</math> eweredig]</p> <p><b>OR/OF</b></p> $\frac{RN}{KP} = \frac{0,75}{1,5} = \frac{1}{2} ; \frac{NM}{PM} = \frac{1}{2} ; \frac{RM}{KM} = \frac{1,25}{2,5} = \frac{1}{2}$ $\therefore \frac{RN}{KP} = \frac{NM}{PM} = \frac{RM}{KM}$ <p><math>\therefore \Delta KPM \parallel \Delta RNM</math> [Sides of <math>\Delta</math> in prop/sye v <math>\Delta</math> eweredig]</p> <p><b>OR/OF</b></p> <p>In <math>\Delta MNR</math>:  <math>1,25^2 = 1^2 + 0,75^2 = 1,5625</math>  <math>\therefore \hat{MNR} = 90^\circ</math> [converse Pyth theorem]</p> <p>In <math>\Delta PKM</math>:  <math>2,5^2 = 1,5^2 + 2^2 = 6,25</math>  <math>\therefore \hat{PKM} = 90^\circ</math> [converse Pyth theorem]</p> $\cos \hat{PKM} = \frac{1,5}{2,5} = \frac{3}{5} \text{ and } \cos \hat{R} = \frac{0,75}{1,25} = \frac{3}{5}$ <p><math>\therefore \hat{PKM} = \hat{R}</math></p> <p>In <math>\Delta KPM</math> and <math>\Delta RNM</math>  <math>\hat{PKM} = \hat{R}</math> [proved]  <math>\hat{P} = \hat{MNR}</math> [proved]  <math>\therefore \Delta KPM \parallel \Delta RNM</math> [<math>\angle</math>; <math>\angle</math>; <math>\angle</math> OR 3<sup>rd</sup> <math>\angle</math>]</p>	<p>✓✓✓ all 3 statements/ <i>al 3 bewerings</i> (3)</p> <p>✓✓✓ all 3 statements/ <i>al 3 bewerings</i> (3)</p> <p>✓ <math>\hat{P} = \hat{MNR}</math></p> <p>✓ <math>\hat{PKM} = \hat{R}</math></p> <p>✓ [<math>\angle</math>; <math>\angle</math>; <math>\angle</math> OR 3<sup>rd</sup> <math>\angle</math>] (3)</p>
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11.2.2	$\hat{P}\hat{K}M = \hat{R}$ $\therefore \hat{P} \text{ is common/gemeen}$ $\therefore \triangle RPQ \parallel \triangle KPM \quad [\angle\angle\angle]$ $\frac{RP}{KP} = \frac{RQ}{KM} \quad [\triangle RPQ \parallel \triangle KPM]$ $\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}$ $\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	<p>✓ S</p> <p>✓ <math>\triangle RPQ \parallel \triangle KPM</math></p> <p>✓ S</p> <p>✓ subst correctly/ korrek</p> <p>✓ <math>RQ = 5\frac{5}{12}</math></p> <p>✓ <math>NQ = \text{answ/antw}</math> (6)</p>
	<b>OR/OF</b>	
	$\hat{R}\hat{N}M = \hat{P}$ $\therefore \hat{R} \text{ is common/gemeen}$ $\therefore \triangle RNM \parallel \triangle RPQ \quad [\angle\angle\angle]$ $\frac{RP}{RN} = \frac{RQ}{RM} \quad [\triangle RNM \parallel \triangle RPQ]$ $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	<p>✓ S</p> <p>✓ <math>\triangle RNM \parallel \triangle RPQ</math></p> <p>✓ S</p> <p>✓ subst correctly/ korrek</p> <p>✓ <math>RQ = 5\frac{5}{12}</math></p> <p>✓ <math>NQ = \text{answ/antw}</math> (6)</p>
	<b>OR/OF</b>	
	<p>In <math>\triangle MNR</math>:</p> $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore \hat{M}\hat{N}R = 90^\circ \quad [\text{converse Pyth theorem}]$ <p>In <math>\triangle PKM</math>:</p> $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90^\circ \quad [\text{converse Pyth theorem}]$ <p>In <math>\triangle MNR</math> and <math>\triangle QPR</math></p> <p><math>\angle R</math> is common</p> $\hat{M}\hat{N}R = \hat{P} = 90^\circ$ $\therefore \triangle MNR \parallel \triangle QPR \quad [\angle\angle\angle]$ $\frac{RP}{RN} = \frac{RQ}{RM} \quad [\triangle RNM \parallel \triangle RPQ]$ $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42 \text{ or } 5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67 \text{ or } 4\frac{2}{3}$	<p>✓ S</p> <p>✓ <math>\triangle MNR \parallel \triangle QPR</math></p> <p>✓ S</p> <p>✓ subst correctly/ korrek</p> <p>✓ <math>RQ = 5\frac{5}{12}</math></p> <p>✓ <math>NQ = \text{answ/antw}</math> (6)</p>
		<b>[10]</b>

**TOTAL/TOTAAL: 149**